HIGH ENERGY SCATTERING FROM THE AdS/CFT CORRESPONDENCE[a]

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We apply the AdS/CFT correspondence to derive expressions for the scattering amplitudes at high energy for gauge theories at strong coupling. A method is proposed based on the computation of correlators of Wilson loop operators by their stringy duals in AdS spaces using either the supergravity (weak field) or classical (minimal surface) approximations.

1 Introduction

Historically, the interpretation of strong interactions in terms of a string theory has raised much hope [1] but was a deceiving adventure. Indeed, while the Veneziano (resp. Shapiro-Virasoro) amplitudes for Reggeon (resp. Pomeron) exchanges were very promising and at the root of the open (resp. closed) string theories, problems arise when looking for the internal consistency of the whole scheme in our 4-dimensional world: a quantum anomaly requires 26 or 10 dimensions, gravitons and zero-mass vectors unavoidably appear in the spectrum of strong interaction states. So a stringy description remains an open problem for QCD_4.

Recently, the proposal of an AdS/CFT correspondence [2] seems to overcome some difficulties met during the last 30 years. In very brief terms (for a extended review, see[3]) the idea is to unify a "microscopic" and a "microscopic" description of a configuration of $N \gg 1$ three-branes in the so-called Type II-B string theory in 10 dimensions.

In the "microscopic" description, the system gives rise to a 4-dimensional $SU(N)$ gauge theory (with $\mathcal{N} = 4$ supersymmetries), while in the "macroscopic" one, it is the source of a gravitational background equipped with a $AdS_5 \otimes S_5$ metric with the physical Minkowski space lying at the boundary of AdS_5. A duality property is conjectured between the 4-dimensional $SU(N)$ gauge theory at *strong* coupling and the gravitational background at *weak* coupling. Interestingly enough, the dynamical rôle of the fifth dimension in AdS_5 is crucial for the validity of the correspondence.

The case of a gauge theory with $\mathcal{N} = 4$ supersymmetries corresponds to a 4-dimensional, non-confining, conformal field theory. The conjecture could

[a] Talk presented at the DIS00 workshop, Liverpool, April 2000.

be enlarged to confining theories without supersymmetry (e.g. see [4]) by introducing a "horizon" scale in the 5-th dimension. The term "horizon" comes from the consideration of a black hole metric in the bulk of AdS space in order to break supersymmetry. Even if the exact dual of QCD_4 has not yet been identified, these dualities give a laboratory framework for gauge observables at strong coupling. For instance, the Wilson area criterion for confinement can be explicitely verified [5].

2 High energy amplitudes: obervables and results

Let us briefly outline the derivation of our papers [6,7]. Scattering amplitudes in the high energy limit (and small momentum transfer) can be conveniently expressed in terms of a correlator of Wilson loops [8].

$$A(s,q^2) = -2is \int d^2x_\perp e^{iqx_\perp} \left\langle \frac{W_1 W_2}{\langle W_1 \rangle \langle W_2 \rangle} - 1 \right\rangle \tag{1}$$

where the two *tilted* Wilson loops follow elongated trajectories along the light-cone direction with transverse separation a and a tilting angle θ around the impact parameter axis. This corresponds to the scattering of colorless quark-antiquark pairs of mass $m \sim a^{-1}$. Indeed, the geometrical parameters of the configuration can be related to the energy scales by analytic continuation $\cos\theta \to \cosh\chi \equiv \frac{1}{\sqrt{1-v^2}} = \frac{s}{2m^2} - 1$ where $\chi = \frac{1}{2}\log\frac{1+v}{1-v}$ is the Minkowski angle (rapidity) between the two lines, and v is the relative velocity.

The results we obtain distinguish between the large and the small impact parameter kinematics. At large impact parameter, we could use [6] the supergravity approximation of the appropriate type II-B string theory, since the fields, in particular gravity, are weak. We computed the exchange contribution of all zero-mode fields between the two separated AdS_5 surfaces whose geometry is fixed by area minimization with the two initial Wilson loops at the boundary. Looking to the contribution of the various fields (dilaton, Kaluza-Klein scalars, antisymmetric tensors mixed with Ramond-Ramond forms and the graviton) we find a hierarchy of real phase-shifts $\delta(b) \equiv \log\left\langle \frac{W_1 W_2}{\langle W_1\rangle\langle W_2\rangle}\right\rangle$ contributing to elastic scattering at large impact parameter only. Indeed, this hierarchy is different from the static Wilson loop correlator [5], since the graviton is dominant and not the Kaluza-Klein scalars. The potential problems with unitarity are avoided, since the weak field approximation appears to be valid only at very large impact parameter $L/a \gg s^{2/7}$ where the scattering amplitudes are purely elastic. Note, however, the retentivity of the gravitational interaction, which is still mysterious in the general context of the AdS/CFT correspondence where the decoupling from gravity is expected.

In a second paper [7], we addressed the problem of small impact parameter and the origin of inelasticity, i.e. imaginary contributions to the phase shift. We concentrated on a situation where the difficulty with supergravity field exchanges does not arise, since there exists a single connected minimal surface which gives the dominant contribution to the scattering amplitude in the strong coupling regime. This allows us to extend our study to small impact parameters, where inelastic channels are expected to play an important rôle. Moreover, it is possible to investigate both cases of conformal (non-confining) and confining cases by considering the appropriate geometries in AdS spaces. Our goal was to understand the rôle of confining geometries in the characteristic features of scattering amplitudes at high energy. The main expected feature is Reggeization, i.e. the determination of the amplitudes by singularities (poles and cuts) in the complex plane of the crossed channel partial waves, moving with $t \equiv q^2$.

Our main result [7] is that high energy amplitudes are governed by the geometry of minimal surfaces, generalizing the *helicoid* in different AdS geometries with the elongated tilted Wilson loops at the boundary. Indeed, the confining geometries have the remarkable properties to admit approximately flat configurations near the horizon scale in the fifth dimension and thus the tilting angle induces (approximate) helicoidal solutions for the minimal surface problem. For this solution and after analytic continuation, one finds a Regge singularity corresponding to a linear double Regge pole trajectory with intercept one

$$\alpha(t) = 1 + \frac{R_0^2}{4\sqrt{2g_{YM}^2 N}} t \ , \tag{2}$$

where R_0 is the horizon scale and g_{YM}, the gauge theory coupling.

The results in confining geometries for impact parameter larger or of the order of R_0 can be contrasted with the conformal (non-confining) $AdS_5 \otimes S_5$ case which, using an asymptotic evaluation (the mathematical knowledge on minimal surfaces embedded in AdS spaces is yet limited!), leads to amplitudes with flat trajectories of the type

$$A(s,t) \sim i s^{1+\frac{2\pi^4}{\Gamma(1/4)^4}\cdot\frac{\sqrt{2g_{YM}^2 N}}{2\pi}} t^{-1-\frac{F(\pi/2)}{2\pi}\ \frac{\sqrt{2g_{YM}^2 N}}{2\pi}} , \tag{3}$$

where $F(\pi/2) \sim .3\pi$ comes from an anomalous dimension computed in [9].

However, even in the confining cases, it may of course happen that the impact parameter distance between the two Wilson lines becomes much smaller than R_0. In this case (see Fig. 1) the minimal surface problem becomes less affected by the black hole geometry and will just probe the small z region of the geometry.

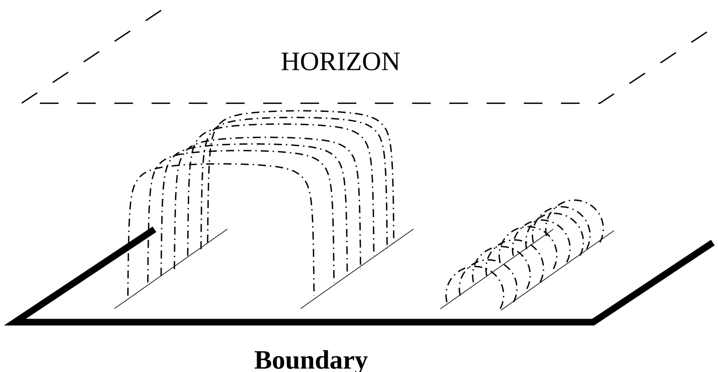

Figure 1: The minimal surface in the black hole geometry. For simplicity the Wilson lines are drawn here with vanishing angle of tilt $\theta = 0$.

The precise behaviour at these shorter distances will depend on the type of gauge theory and, in particular, on the small z limit of the appropriate metric. In fact this limit resembles the original $AdS_5 \times S^5$ geometry. Note that the same behaviour can be equivalently obtained through rescaling, by keeping the impact parameter fixed and putting the scale $R_0 \to \infty$. The conformal behaviour of the amplitude (3) may thus give a hint of the small impact parameter limit also present in the physical confining case of QCD_4, and thus a kind of hard-soft transition in impact parameter.

3 Outlook

Using the AdS/CFT correspondence, we found a relation between high-energy amplitudes in gauge theories at strong coupling and minimal surfaces generalizing the helicoid in various AdS backgrounds. We considered three cases: (i) flat metric approximation of an AdS black hole metric giving rise to Regge amplitudes with linear trajectories, (ii) an approximate evaluation for the conformal $AdS_5 \times S^5$ geometry leading to flat Regge trajectories and (iii) evidence for a transition, in a confining theory, from behaviour of type (i) to (ii) when the impact parameter decreases below the interpolation scale set by the horizon radius. It would be quite useful to supplement the approximations made in our investigations by an evaluation of the string fluctuation pattern around the classical configurations we analyzed, in order to have a more precise determination of the predictions based on the AdS/CFT correspondence. After that, we will be able to discuss the validity and usefulness of this stimulating conjecture in a deeper way.

Acknowledgments

This work was done in tight collaboration with R. Janik whose name should be associated to all the results mentioned in this contribution. This work was supported in part by the EU Fourth Framework Programme 'Training and Mobility of Researchers', Network 'Quantum Chromodynamics and the Deep Structure of Elementary Particles', contract FMRX-CT98-0194 (DG 12 - MIHT).

References

1. For a good review and introduction to strings for strong interactions, see P.H.Frampton, *Dual Resonance Models* , (1974, Benjamin; New edition: 1986, World Scientific).
2. J. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231;
   S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Phys. Lett.* **B428** (1998) 105;
   E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 253.
3. O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Phys. Rept.* **323** (2000) 183.
4. E. Witten, *Adv. Theor. Math. Phys.* **2** (1998) 505;
   S.-J. Rey, S. Theisen and J.-T. Yee, *Nucl. Phys.* **B527** (1998) 171;
   A. Brandhuber, N. Itzhaki, J. Sonnenschein and S. Yankielowicz, *Phys. Lett.* **B434** (1998) 36
5. J. Maldacena, *Phys. Rev. Lett.* **80** (1998) 4859;
   S.-J. Rey and J. Yee, *Macroscopic strings as heavy quarks in large N gauge theory and anti-de Sitter supergravity,* hep-th/9803001.
6. R.A. Janik and R. Peschanski, *Nucl. Phys.* **B565** (2000) 193; R.A. Janik, *Gauge Theory Scattering from the AdS/CFT correspondence* hep-th/9909124, talk presented at the NATO ASI 'Progress in String Theory and M-theory' Cargese 99.
7. R.A. Janik and R. Peschanski, *Minimal surfaces and Reggeization in the AdS/CFT correspondence* hep-th/0003059, to be published.
8. O. Nachtmann, *High Energy Collisions and Nonperturbative QCD,* hep-ph/9609365 (see e.g. eq. (3.87) for colourless state scattering), lectures given at Banz (Germany) 1993 and at Schladming (Austria) 1996.
9. N. Drukker, D.J. Gross and H. Ooguri, *Phys.Rev.,* **D60** (1999) 125006;H. Ooguri, *Prog.Theor.Phys.Suppl.,* **134** (1999) 153.